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Wind Profile Estimation from Point to Point Laser Distortion Data

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Abstract

This report presents the author's results on the problem of using laser distortion data to estimate the wind profile along the path of the beam. A new model for the dynamics of the index of refraction in a non-constant wind is developed. The model agrees qualitatively with theoretical predictions for the index of refraction statistics in linear wind shear, and is approximated by the predictions of Taylor's hypothesis in constant wind. A framework for a potential in-flight experiment is presented, and the estimation problem is discussed in a maximum likelihood context.

1 Introduction

Remote sensing of wind profiles is a problem of interest in both the atmospheric sciences and flight research. Applications such as measuring wind profiles along the space shuttle launch trajectory, detection of microbursts at airports, and confirmation of computational fluid dynamics predictions in flight test require advances in remote sensing methods for wind profiles.

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The use of lasers in remote sensing is now well established. There are two basic methods used. In the laser doppler velocimetry, backscatter from particles, water vapor, or boundary layers is used to measure wind velocity. Tje other method, considered in this research, uses the forward scattering of the laser beam, specifically the distortion of the beam in atmospheric turbulence, to measure the wind profile.

2 Laser Distortion and Remote Sensing

Wave propagation in turbulence was studied by Tatarskii [19] and others [16]. The solutions to Maxwell's equations are assumed to have a sinusoidal time dependence with frequency ω, and the index of refraction fluctuations $n_1(r)$ are assumed to be frozen in time, and are modelled as a Gaussian random field with zero mean and a modified von Karman spectral density. The quantity $V(r)$ satisfying

$$E(r,t) = Re[e^{i(\omega t - kz)}V(r)]$$

obeys the forward scattering equation

$$\frac{\partial}{\partial z}V_z = \frac{i}{2k}\nabla^2 V_z + ikn_{1,z}V_z \tag{1}$$

where z is the distance from the laser along the path of the beam, n_1 is the random index of refraction deviations, k is the wave number, and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. Note that time changes are not represented in Equation 1.

Equation 1 cannot be solved in general, so it is somewhat difficult to work with. Rytov [17] developed a linearization scheme for $\log V$ that is used in most of the remote sensing literature. Let $\Psi = logV$. Then

$$\frac{\partial}{\partial z}\Psi_z = \frac{i}{2k}\nabla^2\Psi_z + \frac{i}{2k}\nabla\Psi \cdot \nabla\Psi_z + ikn_{1,z}$$

Then

$$2Re[\Psi] = \log|V|^2 = \log I$$

and

$$\text{phase}(V) = Im[\Psi]$$

where I is the irradiance, or energy distribution in the beam. The Rytov approximation keeps only the first order perturbation terms for $\Psi = \log V$. Hence let Ψ^0 be the solution to

$$\frac{\partial}{\partial z}\Psi_z^0 = \frac{i}{2k}\nabla^2\Psi_z^0 + \frac{i}{2k}\nabla\Psi_z^0 \cdot \nabla\Psi_z^0 \text{ , } \Psi_0^0 \text{ given}$$

The Rytov approximation for Ψ is the solution to

$$\frac{\partial}{\partial z}\Psi_z = \frac{i}{2k}\nabla^2\Psi_z + \frac{i}{k}\nabla\Psi_z \cdot \nabla\Psi_z^0 - \frac{i}{2k}\nabla\Psi_z^0 \cdot \nabla\Psi_z^0 + ikn_{1,z} \tag{2}$$

The Rytov approximation is valid in weak turbulence, or over distances of less than about 100 m.

For remote sensing of wind velocity, the time behavior of the index of refraction must be taken into account. Hence the index of refraction deviations are now $n_1(r,t)$, and the scattering model will be

$$\frac{\partial}{\partial z}V_{z,t} = \frac{i}{2k}\nabla^2 V_{z,t} + ikn_{1,z,t}V_{z,t} \text{ , } V_0 \text{ given}$$

In most of the remote sensing literature, it is assumed that the index of refraction field is frozen and moving with constant wind velocity U, which is Taylor's hypothesis. Under this assumption,

$$n_1(r,t) = n_1(r + Ut, 0)$$

Remote sensing of the average wind velocity across the beam has been studied by Ishimaru [6], Lawrence [8], Lawrence, Ochs and Clifford [10] and Lee and Harp [13].

Remote sensing of the wind profile has received somewhat less attention. Lee [12] considered spatial filtering across the aperture of both the transmitter and receiver was able to obtain wind profile measurements that agreed well with anemometer data. Little and Ekers [9] and Briggs [3] decompose the index of refraction n_1 in the frequency domain, and assume that at each frequency λ, the index of refraction field is moving with velocity $U(\lambda)$, hence

$$n_1(r,t) = \int_{\mathbf{R}^3} e^{i[\lambda, r + tU(\lambda)]}\hat{n}_1(\lambda)\, d\lambda$$

Another approach has been to partition the atmosphere into layers, each moving with a different, but constant velocity.

3 Problem Statement

There are three problems to be addressed in this study. The first is the construction of a model for the dynamic behavior of the index of refraction in wind shear, or non-homogeneous winds. Clearly, Taylor's hypothesis will not be of use in determining wind profiles, and a new model for the dynamics and statistics of $n_1(r,t)$ is required. Such a model must take into account the interplay between wind shear and the statistics of the index of refraction field, yet still be simple enough for engineering calculations.

The second problem is the design of an experiment to measure wind profiles over a given distance using existing, available technology. The distances of interest are 5 - 10 m, 100 - 500 m, and 1 - 5 km. In this report, the focus will be on distances of about 5 - 10 m.

The third problem is the estimation algorithm for the wind profile. The desired algorithm should be fully automated, and operate with as much speed and as little computing requirements as possible.

4 A Model for the Dynamic Behavior of the Index of Refraction in Atmospheric Turbulence with Wind Shear

In order to use the distortion of a laser beam to measure wind velocity, it is necessary to model the time dependence of the index of refraction field in wind. Usually this is done by assuming that a 'frozen' index of refraction field n is moving with constant uniform velocity U, hence

$$n(t,r) = n(0, r - Ut)$$

We observe the distortion of the phase or magnitude of $V(t,z)$, where V is the solution to

$$\frac{\partial}{\partial z}V(t,z) = \frac{i}{2k}\nabla^2 V(t,z) + ik(n(t,z)-1)V(t,z)$$

Clearly if there is any wind shear, that is the wind velocity is not uniform, and we wish to estimate a wind profile, then a different dynamic model for n is needed.

It is well known that in wind shear, which in this report is considered to be any inhomogeneity in the mean wind velocity (time average), the turbulence field, and hence the index of refraction is no longer isotropic. This effect was studied by Trevino and Laiture [20] in the case of microbursts. The model used must take into account this effect.

Tatarskii [19] showed that index of refraction fluctuations were proportional to fluctuations in the potential temperature, hence we consider a dynamic model for temperature fluctuations.

4.1 A Dynamic Model For Temperature And Velocity Fluctuations

A brief derivation of the dynamics of the temperature and velocity fields is taken from Lumley and Panofsky [14] and is presented in this section. Let T denote temperature. Then

$$T = T_0 + T'$$

where T_0 is the nominal temperature, which takes into account the expected changes in temperature due to altitude, and T' is the fluctuation. We also express T' as

$$T' = \bar{T}' + \theta$$

where $\bar{T}'$ denotes the average value and θ the random fluctuations.

Similarly, let U be the velocity field, with

$$U = \bar{U} + u$$

The dynamic equations for u and θ are

$$\dot{u} = -(\nabla\bar{U})u - (\nabla u)\bar{U} - (\nabla u)u + \overline{(\nabla u)u} + \frac{-1}{\rho_0}\nabla P^{*} + \nu\Delta u + \frac{g}{T_0}\theta(0\ 0\ 1)^{*}$$

$$\dot{\theta} = \kappa\Delta\theta - (\nabla\theta)\bar{U} - \{(\nabla\theta)u - \overline{(\nabla\theta)u}\} - (\nabla\bar{T}')u \qquad (3)$$

where ρ_0 is the nominal density, P is the pressure, ν is the kinematic viscosity, and κ is the thermal diffusivity.

In the next section, the equation for θ is used to derive a stochastic model for the temperature field.

4.2 A Stochastic Model For The Temperature Fluctuations

In the frozen temperature field models, θ is taken to be a locally isotropic and homogeneous gaussian random field with zero mean. Requiring our dynamic model to have the same statistics in the constant wind case $\bar{U}(r) \equiv \bar{U}$ provides some rational for choosing a dynamic model.

In Equation 3 the last term represents a transfer of energy from the mean temperature gradiant by means of turbulence. The third term represents the transport of the temperature fluctuations by the turbulence, with a correction for the mean.

The main idea of our model is to represent the third and fourth terms as a white noise in time and an isotropic homogeneous random field in $r \in \mathbf{R}^3$. Hence our model is

$$\dot{\theta}_t = \kappa \Delta \theta_t - (\nabla \theta_t)\bar{U} + N_t \tag{4}$$

In Equation 4 the first term describes the diffusion of heat, the second term describes transport of the temperature field by the mean wind, and the last term describes the disturbence of the temperature field due to turbulence. The 'in time' correlation of the third and fourth terms of Equation 3 is not, of course, that of white noise, but because $\kappa << 1$ the system described in Equation 4 is a low pass filter, hence the white noise assumption is reasonable. The model is now completely specified, except for the covariances of θ_0 and N_t.

Suppose for a moment that there is no wind. If a steady state covariance for θ exists call it R, and its spectral density P. Let B be the covariance predicted by the frozen field model and Φ the corresponding spectral density, with $\Phi(0) < \infty$. R_t obeys the equation

$$\dot{R}_t(r_1 - r_2) = \kappa(\Delta_1 + \Delta_2)R_t(r_1 - r_2) + R_N(r_1 - r_2)$$

where R_N is spatial covariance for N with corresponding spectral density P_N. Since N is spatially homogenous, θ will be also, hence we can write

$$\dot{R}_t(r) = 2\kappa \Delta R_t(r) + R_N(r)$$

If we take the spatial spectral density for N to be

$$P_N(\lambda) = 2\kappa |\lambda|^2 \Phi(\lambda)$$

then there will be a steady state covariance R, with spectral density

$$P(\lambda) = \Phi(\lambda)$$

or $R = B$. Hence, in the no wind case, the spatial covariance agrees with that for the frozen field model.

If we assume that $\bar{U}$ is a nonzero constant, we again have $P(\lambda) = \Phi(\lambda)$. Hence this model agrees with the frozen field model for constant wind, at least when looking at only the spatial covariance.

Let S_t be the semigroup generated by $\kappa\Delta$ on the Banach space of Fresnel class functions on $\mathbf{R}^3$. Finally, assume that the wind is constant, $\bar{U}(r) \equiv \bar{U}$. The space-time covariance for θ is then:

$$E[\theta(t,r)\theta(t',r')] = [S(t-t')B](r - \bar{U}(t-t') - r') \ , \ t > t' \tag{5}$$

In the frozen field model, this covariance is $B(r - \bar{U}(t-t') - r')$, which is the prediction of Taylor's Hypothesis. Since Taylor's hypothesis is widely applicable, even in the presence of some wind shear, it is important for our model not to deviate from it too drastically. In our case, the thermal diffusivity is

$$\kappa \approx 5 \times 10^{-8} m^2/s$$

hence our covariance for θ should not deviate too much from Taylor's hypothesis in constant wind.

4.3 Covariance Calculations For More General Winds

The case of primary interest in this discussion is when the wind $\bar{U}$ is not a constant, but varies in space. We assume that the flow is incompressible, hence $\nabla \cdot \bar{U} = 0$. In this case, the covariance for θ obeys

$$\dot{R}_t(r_1,r_2) = \kappa(\Delta_1+\Delta_2)R_t(r_1,r_2) - \nabla_1 R_t(r_1,r_2)\bar{U}(r_1) - \nabla_1 R_t(r_1,r_2)\bar{U}(r_2) + R_N(r_1-r_2) \tag{6}$$

It is difficult to proceed further with this unless a special form is assumed for $\bar{U}(r)$. It is known that a linear wind shear results in a loss of isotropy in the turbulence field, and we expect the same for temperature. Let

$$\bar{U}(r) = U_0 + U_1 r \tag{7}$$

In this case the covariance for θ is translation invariant, so

$$R_t(r_1, r_2) = R_t(r_1 - r_2)$$

and R_t obeys

$$\dot{R}_t(r) = 2\kappa\Delta R_t(r) - \nabla R_t(r) U_1 r + R_N(r) \tag{8}$$

Let T_t be the semigroup generated by $2\kappa\Delta - U_1 r \cdot \nabla$ on the Fresnel class functions. Then

$$[T_t f](r) = E[f(e^{-U_1 t} r + x_t)]$$

where x_t is a Gaussian random vector in $\mathbf{R}^3$ with zero mean and covariance

$$E[x_t x_t^*] = M_t$$

$$M_t = 4\kappa \int_0^t e^{-U_1 s} e^{-U_1^* s}\, ds$$

If there is a steady state covariance in this case, it will be

$$\begin{aligned}
R(r) &= \int_0^\infty T_t R_N\, dt \\
&= \int_0^\infty \int_{\mathbf{R}^3} E[exp(i[\lambda, e^{-U_1 t} r + x_t])P_N(\lambda)\, d\lambda\, dt \\
&= \int_0^\infty \int_{\mathbf{R}^3} exp(i[e^{-U_1^* t}\lambda, r] - \frac{1}{2}[M_t\lambda, \lambda])P_N(\lambda)\, d\lambda\, dt \\
&= \int_0^\infty \int_{\mathbf{R}^3} exp(i[\lambda, r]) exp(-2\kappa \int_0^t |e^{U_1^* s}\lambda|^2\, ds) P_N(e^{U_1^* t}\lambda)\, d\lambda\, dt
\end{aligned}$$

the last equation being obtained by a change of variable, and the fact that

$$|e^{U_1^* t}| = exp(t \cdot tr\ U_1^*) = 1$$

by our incompressability assumption. Hence we can write the steady state spectral density as

$$P(\lambda) = 2\kappa \int_0^\infty exp(-2\kappa \int_0^t |e^{U_1^* s}\lambda|^2\, ds) |e^{U_1^* t}\lambda|^2 \Phi(e^{U_1^* t}\lambda)\, dt \tag{9}$$

The integral exists for $\lambda \neq 0$ since the function

$$exp(-2\kappa \int_0^t |e^{U_1^* s}\lambda|^2\, ds)\ 2k|e^{U_1^* t}\lambda|^2$$

is a probablility density function in t, and Φ is a bounded function.

Note that isotropy is lost except at the higher frequencies. This agrees with the observations of Lumley and Panofsky [14].

The model presented above is, in the belief of the author, an improvement on Taylor's hypothesis, which ignores time changes in the temperature and velocity fields not due to wind velocity, and the effects of wind shear. For constant wind, the model agrees well with Taylor's hypothesis. This model is represented as a state space linear system, and there is a great deal of literature on the estimation of parameters for such systems.

5 Proposals for the Design of an Experiment

In this section, concepts for a remote sensing experiment are presented and discussed.

The distance over which the laser propagates is critical to the type of sensors needed. For example, the irradiance distortion at 5 or 10 m is very small, and at this distance it is better to measure phase distortion. Over larger distances, it may be more convenient (cheaper) to measure the irradiance, or amplitude distortion due to index of refraction fluctuations.

In this study, wind profiles over 5 - 10 m were of particular interest, hence an experimental setup for this case is presented.

It is well known that atmospheric turbulence has dramatic effects on the performance of heterodyning receivers, which are sensitive to phase distortion (see Leader [11]). A concept for an experimental setup is given in Figure 1. The distorted beam is compared with a local oscillator by means of a heterodyning interferometer, and a beam splitter.

For in flight experiments, the setup in Figure 1 is impractical because of the massive, voluminous, delicate equipment needed for the receiver. For an in flight experiment, it might be desireable to measure the flow over a wing, and the setup in Figure 1 would require putting a laser and interferometer on the wing tip, which is clearly infeasible. Another posibility is to locate the laser and interferometer close together and bend the beam back using a mirror. Laser beams with straight and 'folded' paths have been compared by Smith and Pries [18]. Distortion of beams with folded paths using corner reflectors and flat mirrors have been studied by Gamo, Jagannathan and Majumdar [4].

Unfortunately, it is necessary to use a flat mirror for wind velocity measurements. A corner reflector would invert the beam, so on the first half of the folded path, the wind appears to move in one direction, and on the second half appears to move the other way. Some sort of active control of the flat mirror and the laser will be required due to vibration of the wing. An experimental setup using a such a folded path is shown in Figure 2.

Over longer distances, less expensive receivers that detect the irradiance can be used in configurations similar to those in Figures 1 and 2 without the need for a reference beam, or local oscillator.

6 Estimation Algorithm

In this section, an approach for maximum likelihood estimation of the wind profile is discussed. There are many advantages to using maximum likelihood to estimate parameters for dynamic systems, including efficiency and consistency of the estimates whenever these conditions can be satisfied. The models described above for the index of refraction and laser distortion describe a linear state space system with a non-linear observation. The 'plant' model is

$$\dot{n}_{1,t} = \kappa \Delta n_{1,t} + \nabla n_{1,t} \cdot U + N_t \tag{10}$$

where the function $U(r)$ is the unknown wind profile. The observation equation can be written as

$$v(t) = C(n_{1,t}) + N_t^o \tag{11}$$

where the form of C depends on our experimental setup, and whether approximations to the forward scattering equation, such as the Rytov approximation, are used. N^o is the observation noise, and is assumed to be a white Gaussian noise.

The problem of estimating the wind profile is now in the form of estimating an unknown distributed parameter of a linear dynamic system from a non-linear observation plus noise.

There is a great deal of literature on the estimation of parameters of linear dynamic systems with linear observations, such as Maine and Iliff [15]. This problem has also been considered for infinite dimensional , or distributed parameter systems, such as we have here [2, 7]. A dated, but still useful review is found in Goodson and Polis [5]. In general, this is still an open research problem.

The observation equation can be linearized using the Rytov approximation. Suppose the experiment measures the phase distortion of the beam at m points. Then the observation can be approximated by

$$C_{\mathbf{n}_1} = \int_{\mathbf{R}^3} K(r) n_1(r)\, dr$$

where $K(r)$ takes on values in $\mathbf{R}^m$. Let S_t be the semigroup generated by $\kappa\Delta + U \cdot \nabla$ on the fresnel class functions $\mathcal{F}$. Then the covariance of v is

$$I + R$$

Where

$$R(t,s) = \int \int K(r)[S_t R_n S_s^*](r,r') K^*(r')\, dr\, dr'$$

where R_n is the steady state spatial covariance of n_1. In the constant wind case, R_n corresponds to the modified Von Karman spectral density.

One possibility is to use the Kalman filter to calculate the likelihood ratio for v given U with respect to v with no signal. The result is (Balakrishnan [1])

$$\Lambda(U) = e^{\int_0^t [C\hat{n}_{1,s}, v(s)] - \frac{1}{2}|C\hat{n}_{1,s}|^2 - \frac{1}{2} Tr.CP(s)C^*\, ds} \tag{12}$$

where $\hat{n}_1$ is the Kalman filter estimate for n_1 and P is the corresponding error covariance. Both $\hat{n}_1$ and P will be functions of U.

This method presents difficulties from a computational perspective, because it requires the estimation of an infinite dimensional state (or large finite dimensional approximation), and solution of an operator valued Riccatti equation for each iteration. This is very expensive in computation time, and may have numerical stability problems as well.

One solution to this dilemma is to make the very realistic assumption that we receive sampled data

$$v_n = Cn_{1,n\Delta} + N_n^o$$

hence at time step n we have $n \times n$ data points that form a Gaussian random vector with zero mean and a covariance dependent on U. Let $v^n = \{v_1, v_2 \cdots v_n$. Then a direct approach can be taken to solve

$$\max_U p(v^n|U)$$

Note that this will involve assuming some form for U, such as a p'th order polynomial, with the incompressibility constraint,

$$\nabla \cdot U = 0$$

or a piecewise linear form.

7 Conclusion

The problem of estimating wind profiles from laser distortion measurements was considered in this report. A suitable index of refraction model was developed and presented. Two concepts for an experiment were presented and the resulting estimation problem was discussed in a maximum likelihood context. Further research is required to design the hardware and optics for the experiment and to develop a maximum likelihood estimation algorithm for the model presented. Both of these are important research problems at this point in time.

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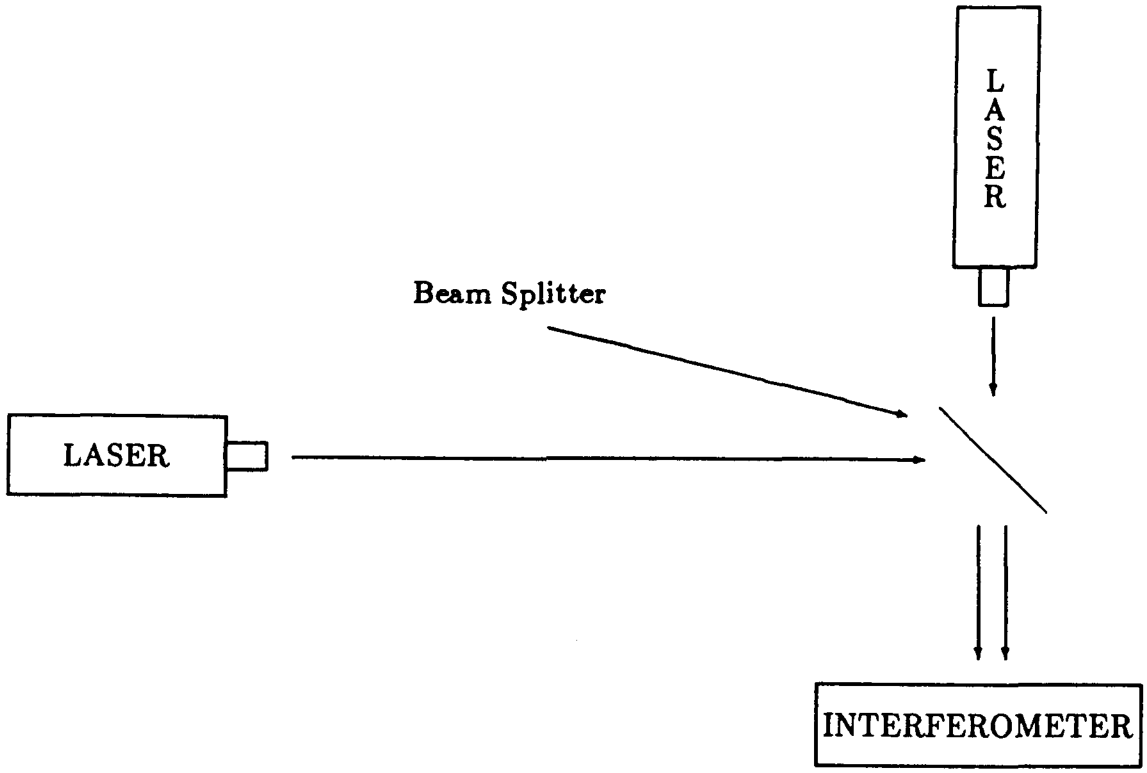

Figure 1: Wind Profile Measurement Experiment for Distances Under 10 m.

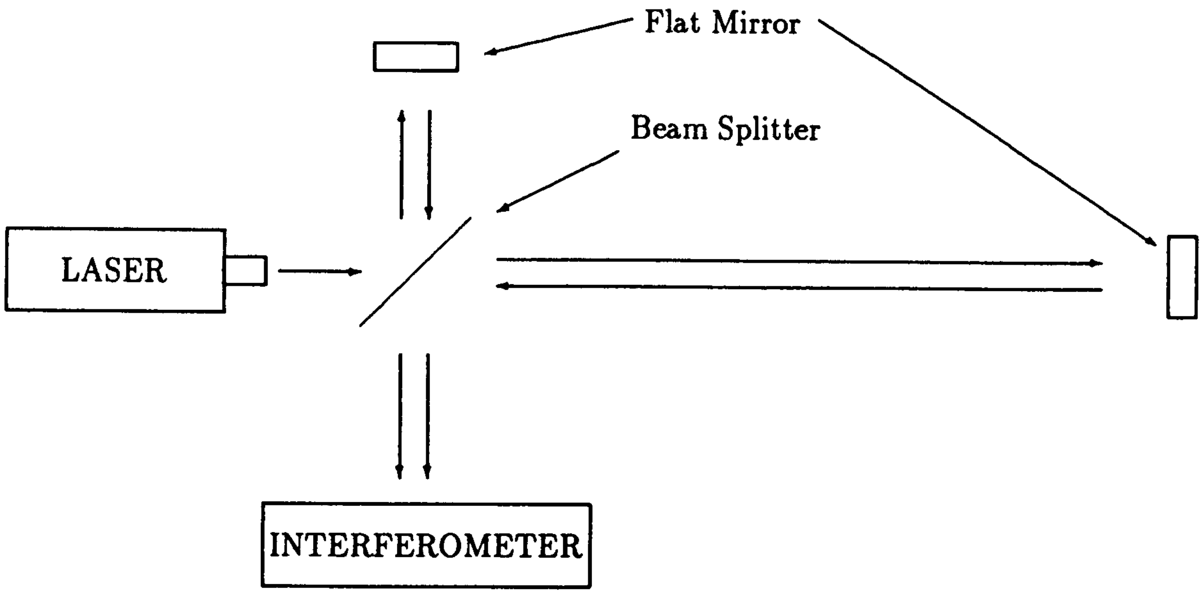

Figure 2: Wind Profile Measurement Experiment for Distances Under 10 m with Folded Path